Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

MONETARY POLICY
SOLUTIONS TO AUGUST 19 EXAM, 2016

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple cash-in-advance model with only consumption in the utility function, there is no unique optimal monetary policy.

A True. Various monetary policies leading to various inflation rates will have no effect on output and consumption as superneutrality prevails. As only consumption matters for welfare, any inflation rate is as good as the other. So no unique optimal policy exists.

(ii) In the Lucas "islands" model, an unanticipated aggregate money shock has no real effects as agents have rational expectations.

A False. It is indeed the unanticipated nature of the shock that causes the confusion, which ultimately causes real effects. With asymmetric information, agents on each island observe the money shock, but do not know whether it is an aggregate shock (in which case they should do nothing) or a local shock (in which case they should change labor supply). They solve a signal extraction problem, where they rationally put some weight to the possibility that it is a local shock. As alle agents do the same, aggregate labor supply and output change under rational expectations.

(iii) Under a nominal interest-rate targeting procedure, monetary policymaking performed without knowledge of the realizations of current shocks can be improved by using the money stock as an intermediate target whenever money-market shocks are predominant in the economy.

A False. When money-market shocks are predominant, the movements in the observable money stock will be relatively uninformative about shocks that affects output and inflation. Hence, adjusting the interest rate in response to money stock movements will not improve monetary policy.

QUESTION 2:

Strict inflation targeting and nominal interest-rate rules

Consider the following model for output and inflation determination in a closed economy:

$$\begin{aligned} y_t &= \theta y_{t-1} - \sigma\left(i_{t-1} - \mathrm{E}_{t-1}\pi_t\right) + u_t, \qquad 0<\theta<1, \quad \sigma>0, && (1)\\ \pi_t &= \pi_{t-1} + \kappa y_t + e_t, \qquad \kappa>0, && (2) \end{aligned}$$

where y_t is log of output in period t, i_t is the nominal interest rate (the monetary policy instrument), π_t is the inflation rate, u_t and e_t are independent, mean-zero, serially uncorrelated shocks. E_j is the rational expectations operator conditional on information up to and including period j. It is assumed that $\sigma\kappa < 1$. The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2}\mathrm{E}_t\sum_{j=1}^{\infty}\beta^j\pi_{t+j}^2, \qquad 0<\beta<1.$$

(i) Derive the optimal interest rate rule for i_t as a function of π_t and y_t. (Hint: Treat $\mathrm{E}_t y_{t+1} \equiv y_{t+1} - u_{t+1}$ as the policy instrument, and solve the maximization problem by dynamic programming treating π_t as the state variable; i.e., we have $v(\pi_t) = \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t\left\{-\frac{1}{2}(\pi_{t+1})^2 + \beta v(\pi_{t+1})\right\}$. Then find the optimal policy as $\mathrm{E}_t y_{t+1} = -B\pi_t$, where B is a parameter to be found, and use (1) and (2) to derive the associated nominal interest rate.)

A Using the hint, the relevant value function becomes

$$\begin{aligned} v(\pi_t) &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t\left\{-\frac{1}{2}\left(\pi_t + \kappa y_{t+1} + e_{t+1}\right)^2 + \beta v\left(\pi_t + \kappa y_{t+1} + e_{t+1}\right)\right\},\\ &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t\left\{\begin{array}{l} -\frac{1}{2}\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + e_{t+1}\right)^2 \\ +\beta v\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + e_{t+1}\right) \end{array}\right\}. \end{aligned}$$

The first-order condition is

$$\begin{aligned} &-\mathrm{E}_t\kappa\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + e_{t+1}\right)\\ &+\mathrm{E}_t\beta\kappa v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + e_{t+1}\right)\\ =\ & 0, \end{aligned}$$

$$-\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right)+\mathrm{E}_t \beta v'\left(\pi_t+\kappa\left[\mathrm{E}_t y_{t+1}+u_{t+1}\right]+e_{t+1}\right)=0.$$

Using the Envelope Theorem one gets:

$$\begin{aligned}
v'\left(\pi_t\right) &= -\mathrm{E}_t\left(\pi_t+\kappa\left[\mathrm{E}_t y_{t+1}+u_{t+1}\right]+e_{t+1}\right)+\mathrm{E}_t \beta v'\left(\pi_t+\kappa\left[\mathrm{E}_t y_{t+1}+u_{t+1}\right]+e_{t+1}\right), \\
&= -\left(\pi_t+\kappa \mathrm{E}_t y_{t+1}\right)+\mathrm{E}_t \beta v'\left(\pi_t+\kappa\left[\mathrm{E}_t y_{t+1}+u_{t+1}\right]+e_{t+1}\right)
\end{aligned}$$

implying that

$$v'\left(\pi_t\right)=0.$$

Hence, $\mathrm{E}_t y_{t+1}=-\frac{1}{\kappa}\pi_t$, showing that $B=\kappa^{-1}$. We also have that

$$\begin{aligned}
\mathrm{E}_t y_{t+1} &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t\left[\pi_t+\kappa y_{t+1}\right]\right), \\
\mathrm{E}_t y_{t+1}(1-\sigma\kappa) &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t \pi_t\right),
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}_t y_{t+1}(1-\sigma\kappa) &= \theta y_t-\sigma\left(i_t-\mathrm{E}_t \pi_t\right) \\
\mathrm{E}_t y_{t+1} &= \frac{\theta}{(1-\sigma\kappa)} y_t-\frac{\sigma}{(1-\sigma\kappa)}\left(i_t-\mathrm{E}_t \pi_t\right).
\end{aligned}$$

So, the interest-rate rule follows from

$$-\frac{1}{\kappa}\pi_t=\frac{\theta}{(1-\sigma\kappa)} y_t-\frac{\sigma}{(1-\sigma\kappa)}\left(i_t-\mathrm{E}_t \pi_t\right),$$

$$-\frac{(1-\sigma\kappa)}{\kappa}\pi_t=\theta y_t-\sigma\left(i_t-\pi_t\right),$$

$$\sigma i_t=\left[\sigma+\frac{(1-\sigma\kappa)}{\kappa}\right]\pi_t+\theta y_t,$$

as

$$i_t=\left[1+\frac{(1-\sigma\kappa)}{\sigma\kappa}\right]\pi_t+\frac{\theta}{\sigma} y_t.$$

(ii) Comment on the coefficient on π_t in the optimal interest rate rule, with special emphasis on how its value affects the stability properties of the model.

A The main issue is that the coefficient is greater than one. Hence, it is an active Taylor-type rule, such that any rise in inflation is met by a larger increase in the nominal interest rate. This increases the real interest rate, and will serve to contract output and thus reduce inflation. Hence, it serves a stabilizing role.

(iii) Discuss how the coefficients on π_t and y_t in the optimal interest-rate rule depend on the underlying parameters of the model. Discuss in particular what the parameters reveal about the strict inflation-targeting preferences of the central bank.

A It can be seen that the structural parameters σ and κ reduce the inflation coefficient. This is because when these values are higher, a smaller nominal interest rate rensponse is needed to stabilize inflation (as demand is more sensitive and inflation is more sensitive to demand). Furthermore, it is observed that output increases will lead to nominal interest rate changes, even though the central bank is conducting strict inflation targeting. The reason being that output changes provides information about inflation one period ahead (as long as there is output inertia; i.e., when $\theta > 0$). Hence, the parameter values, and the variables in the interest rate rule, tell little about the preferences of the central bank.

QUESTION 3:

Rogoff conservativeness in a New-Keynesian Model?

Consider a one-equation variant of a New-Keynesian model of inflation determination:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is inflation, $0 < \beta < 1$ is a discount factor, E_t is the rational expectations operator, $\kappa > 0$ is a parameter, x_t is the output gap, and e_t is a “cost-push” shock that follows the process

$$e_t = \rho e_{t-1} + \widehat{e}_t, \qquad 0 < \rho < 1,$$

where $\widehat{e}_t$ is a mean-zero i.i.d. disturbance. It is assumed that the monetary policymaker controls x_t with the aim of maximizing the discounted sum of

$$U_t = -\frac{\lambda}{2} x_t^2 - \frac{1}{2} \pi_t^2, \qquad \lambda > 0. \tag{2}$$

(i) Show that under discretionary policymaking, optimal policy is characterized by

$$-\lambda x_t = \kappa \pi_t. \tag{3}$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive “cost-push” shock.

A Under discretion, expectations cannot be affected by policy, so maximizing

$$-\frac{1}{2}\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\lambda x_{t+i}^2+\pi_{t+i}^2\right],\qquad 0<\beta<1,$$

w.r.t. x_t subject to (1) is equivalent of maximizing

$$-\frac{\lambda}{2}x_t^2-\frac{1}{2}\pi_t^2+F_t,$$

w.r.t. x_t subject to

$$\pi_t=\kappa x_t+f_t,$$

taking as given F_t and f_t. This immediately provides the first-order condition:

$$-\lambda x_t=\kappa\pi_t.$$

It describes a "leaning against the wind" policy. If inflationary pressures arise due to a positive "cost-push" shock, the policymaker should contract output ($x_t<0$) such that the marginal cost of lower output equals the marginal gain of reducing inflation.

(ii) Assume now that the policymaker can commit to a policy of the form

$$x_t^c=-\omega e_t, \tag{4}$$

where ω is a policy-rule parameter. Derive the optimal relationship between x_t^c and π_t^c. [Hint: Combine (4) with (1) to show that $\pi_t^c=[\kappa/(1-\beta\rho)]\,x_t^c+[1/(1-\beta\rho)]\,e_t$ and maximize U_t w.r.t. x_t^c subject to this expression for π_t^c.]

A Use the hint. Inflation follows from the Phillips curve (1), together with the policy rule (4), as:

$$\begin{aligned}\pi_t^c &= \beta\mathrm{E}_t\pi_{t+1}^c+\kappa x_t^c+e_t,\\ &= \beta\mathrm{E}_t\pi_{t+1}^c-\kappa\omega e_t+e_t.\end{aligned}$$

Solving forward:

$$\begin{aligned}\pi_t^c &= \mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[-\kappa\omega e_{t+i}+e_{t+i}\right],\\ &= \sum_{i=0}^{\infty}\beta^i\left[-\kappa\omega+1\right]\rho^i e_t,\\ &= \frac{1-\kappa\omega}{1-\beta\rho}e_t,\end{aligned}$$

or,

$$\begin{aligned}\pi_t^c &= -\frac{\kappa}{1-\beta\rho}\omega e_t + \frac{1}{1-\beta\rho}e_t \\ &= \frac{\kappa}{1-\beta\rho}x_t^c + \frac{1}{1-\beta\rho}e_t\end{aligned}$$

Then maximize

$$-\frac{1}{2}\left[\lambda\left(x_t^c\right)^2 + \left(\pi_t^c\right)^2\right]$$

w.r.t. subject x_t^c to subject to the expression for π_t^c. This gives the first-order condition:

$$\lambda x_t^c + \frac{\kappa}{1-\beta\rho}\pi_t^c = 0.$$

Or written like in the discretionary case:

$$-\lambda x_t^c = \frac{\kappa}{1-\beta\rho}\pi_t^c.$$

(iii) Discuss, based on the result of (ii), whether appointing a "conservative" policy-maker, one characterized by $\lambda^c < \lambda$, is beneficial. Comment on whether $\rho > 0$ is crucial for the answer.

A One can rewrite the relationship found in (ii) as

$$-\lambda\left(1-\beta\rho\right)x_t^c = \kappa\pi_t^c,$$

or,

$$-\lambda^c x_t^c = \kappa\pi_t^c,$$

where

$$\lambda^c \equiv \lambda\left(1-\beta\rho\right) \leq \lambda.$$

It is seen that as long as $\rho > 0$, the commitment solution is consistent with $\lambda^c < \lambda$, i.e., a smaller weight on output than in the social utility function. Hence, Rogoff-conservatism is optimal. It is, however, crucial that $\rho > 0$, as this implies that a current shock has implications for the future. And in this model with forward-looking expectations it is the ability to affect future expectations that is the benefit of commitment. If $\rho = 0$, the future is not affected by current shocks and there is no need to try to affect future expectations by acting conservative with the simple form of commitment shown in (4). But if the shock persists, being conservative implies a tougher stance on future inflation, which helps stabilize current inflation better.